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Abstract

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It is usually assumed in modern theories of nucleosynthesis that the initial composition of the galaxy was pure hydrogen. The large solar and stellar content of helium has appeared to be a difficulty for such an assumption. This question is examined in this paper. Numerical studies are made of the time changes in the compositions of stars and the interstellar medium as a result of stellar evolution. It is concluded that the large helium content of the sun and recently-formed stars can only be produced as a result of the evolution of stars of approximately solar mass. Hence the initial hydrogen hypothesis requires a large age for the galaxy ($\gtrsim 2 \times 10^{10}$ years). The content of long-lived radioactivities in the interstellar medium is also followed as a function of time. It is found that the ratios of radio-activities are very insensitive functions

author

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of time and are approximately those observed in the solar system at the time the helium content is satisfactory. However, it is also concluded that these ratios give little useful information about cosmochronology.

I. Introduction

Modern theories which attribute the origin of the elements to nuclear reactions in stars derive their strength from the observations that the more abundant products of such nuclear reactions are also the more abundant nuclei in nature (Burbidge, Burbidge, Fowler, and Hoyle 1957; Cameron 1957). Because all of the nuclei observed in nature can thus be formed, it has usually been assumed that the gas from which the galaxy formed was initially composed entirely of hydrogen.

The product formed in the first stage of nucleosynthesis is helium. The initial composition of the sun apparently contained about 24 per cent of helium (Gaustad 1964), while the O and B stars formed more recently contain about 35 per cent helium (Aller 1961). This indicates that the material forming the sun and more recent stars has undergone a large amount of nuclear processing in stellar interiors if the galaxy was initially composed of pure hydrogen.

For some time it has been argued that the bulk of this nuclear processing would have to take place during the very early history of the galaxy (Burbidge <u>et al</u> 1957; Taylor and Hoyle 1964). The essentials of this argument are as follows.

One assumes that the sun has a helium content typical of the galaxy as a whole and calculates the energy release in a galactic mass of gas for the coversion of about one quarter of the hydrogen into helium. It is then noted that this energy release is an order of magnitude greater than the present energy output of all the stars in the galaxy, multiplied by a reasonable galactic age. However, the present interstellar medium is only a very small fraction of the mass of the galaxy, and if the galactic age is large then the mass of stars formed from the interstellar medium in the last 5×10^9 years will also be relatively small. Clearly the matter requires closer consideration.

Furthermore, recent studies of stellar evolution indicate that large amounts of helium could not have been produced by a postulated early concentration of O and B stars. Hayashi (1964) has followed the evolution of massive stars through the hydrogen and helium-burning phases. He finds that in advanced stages of evolution the more massive stars have only a thin layer of helium between the hydrogen and helium-burning shell sources (from 3 to 5 per cent of the stellar mass). The helium-exhausted core has a much larger mass. This leads to the expectation that when such stars undergo supernova explosions they will eject only small amounts of helium and comparable amounts of heavier elements. Thus one cannot appeal to the formation of large numbers of O and B stars in the early history of the galaxy to solve the helium problem.

This paper presents the results of a study in which an attempt is made to follow, numerically, the changes in the compositions of stars and the interstellar medium, which take place as a result of stellar evolution and the interchange of gas between the stars and the interstellar medium. There are many uncertainties in our knowledge of the stellar parameters which must be assumed in this study, and hence the study should be regarded only as a reconnaissance of the problem using reasonable values of these parameters.

It will become evident that this problem contains most of the elements of the basic problem of cosmochronology. The solar system contained certain ratios of radioactivities when it was formed. These isotopes have different half-lives, and hence they will have had different histories as they have been produced in stellar interiors, mixed into the interstellar medium, and perhaps have been circulated through the interiors of subsequent generations of stars. Fowler and Hoyle (1960) and Cameron (1962) have discussed possible cosmochronological histories of the galaxy. Their models were necessarily very crude, since they related the buildup of radioactivities and their subsequent decay in the interstellar medium to rates of star formation in the galaxy, generally assumed to be uniform, sudden, or involving an exponential variation with time. These studies yielded rather different solutions for the time onset of nucleosynthesis in the galaxy.

It is evident that a satisfactory solution to the basic problem of cosmochronology must also attempt to follow, numerically, the production of radioactivities in the couse of stellar evolution and their subsequent mixing into the interstellar medium, temporary storage in stars, and decay in either location. It becomes a test of any model for the rate of star formation in the galaxy to determine whether the solar system, when formed, would have a suitable ratio of radioactivities as well as a satisfactory helium content. Hence cosmochronological calculations were carried out in parallel to the composition calculations in this study.

In carrying out this study it was necessary to make assumptions about the stellar luminosity birth rate function, stellar evolutionary lifetimes, the composition distribution at the end of the evolutionary lifetime, the amount of material returned to the interstellar medium, and the rates of production of radioactivities. These are functions of stellar mass. The assumptions are discussed in the following sections.

II. The Rate of Star Formation

In order to describe the gross effects of stellar evolution on the compositional history of the galaxy, the following assumptions have been made:

(a). The galaxy, in all periods of history, is assumed structureless and homogeneous, and any effects of stellar evolution are immediately felt throughout its volume.

(b). The fractional mass of the interstellar medium at any time is completely determined by an assumed prescription. Various prescriptions have been used and will be discussed later.

(c). Any material given off by stars at the end of their lifetimes is immediately mixed with the interstellar medium and that mixture is then used for further star formation.

(d). The initial luminosity function (or the birth rate function) is independent of time in the sense that for any time interval dt_2 the ratio of stars formed of mass $\mathfrak{M}$ in the mass interval $d\mathfrak{M}$ to the total of stars formed of all masses in the same time interval is independent of the time or the rate of formation.

(e). The lifetime of a star of given mass is independent of time. Thus, we ignore the effects on lifetimes of composition differences resulting from stellar evolution.

(f). At time zero the galaxy is completely gaseous and composed of pure hydrogen.

Following the above we let $\mathfrak{M}_g(t)$ denote the mass of the interstellar medium remaining at time t, and $\mathfrak{M}_s(t)$ denote the total mass of stars formed up to time t, so that

$$\mathfrak{M}_g(t) = \mathfrak{M}_g(0) - \mathfrak{M}_s(t) + \mathfrak{M}_e(t) \qquad (1)$$

where $\mathfrak{M}_e(t)$ is the total mass of gas ejected by all evolved stars up to time t. $\mathfrak{M}_g(0)$ is the total galactic mass. By assumption (b), $\mathfrak{M}_g(t)$ is known, so that only $\mathfrak{M}_e$ and $\mathfrak{M}_s$ remain to be determined in order to specify the evolutionary turnover of gas through galactic history. To do this we obtain a second relation between $\mathfrak{M}_e$ and $\mathfrak{M}_s$ as follows:

Let $\Psi(M_V)$ be the initial luminosity function as a function of visual magnitude M_V, which is assumed to be known between the magnitudes -5 and +20. (A specific discussion of these data will be given in Section III.) Below magnitude -5 no stable stars are assumed to exist. However, low mass stars with magnitudes greater than +20 are assumed to exist, but no data about them are available. To account for the mass contained in these stars we follow Salpeter (1955) and define the time-independent quantity F by

$$F \int_{-\infty}^{\infty} \Psi(M_V)\mathfrak{M}(M_V)\,dM_V = \int_{-5}^{20} \Psi(M_V)\mathfrak{M}(M_V)\,dM_V \ ; \qquad (2)$$

where $\mathfrak{M}(M_V)$ is the mass of a star of magnitude M_V.

With the aid of F we define the "normalized" initial luminosity function per unit mass by

$$k(M_V) = \frac{F\Psi(M_V)}{\int_{-5}^{20} \Psi(M_V)\mathfrak{M}(M_V)\,dM_V} \tag{3}$$

so that

$$F = \int_{-5}^{20} k(M_V)\mathfrak{M}(M_V)\,dM_V \tag{4}$$

It then follows that the number of stars of magnitude M_V in dM_V created at time t in dt is

$$k(M_V)\ \mathfrak{M}(M_V)\ d\mathfrak{M}_s\ dM_V \tag{5}$$

To compute the mass of gas ejected by all stars at the end of their lives we define $\tau(M_V)$ as their lifetime and $\mathfrak{M}_r(M_V)$ as the mass of star remaining as a white dwarf or supernova remnant after the gas is ejected. A straightforward calculation then yields

$$\mathfrak{M}_e(t) = \int_{-5}^{20} \int_{0}^{t-\tau(M_V)} k(M_V) \left[\mathfrak{M}(M_V) - \mathfrak{M}_r(M_V)\right] \frac{d\mathfrak{M}_s(t')}{dt'}\,dt'\ dM_V \tag{6}$$

where the upper limit on the time integral is zero if $\tau >> t$ and t' is the integration variable in time between the limits zero and $t-\tau$. The solution of the coupled equations (1) and (6) is then the solution desired.

An analytical solution is impossible, and numerical techniques were used. With a suitable numerical procedure it is not difficult to extend the analysis to include a detailed accounting of the major material constituents, hydrogen, helium, and the remaining class of heavy elements, as a function of time. The details of their production in stellar interiors and their subsequent ejection will be covered in Section IV.

The form of the function $\mathfrak{M}_g(t)$ is the major variable in this model since it governs, for the most part, the rate of stellar formation as a function of time. Its form will be taken as a decreasing exponential or modified exponential to conform with the deductions of Eggen (1962), Schmidt (1959, 1963), Salpeter (1959), and Wilson (1964) that the average rate of star formation in the past is much greater than the present rate. The first three authors assume that the rate of formation has been decreasing monotonically since time zero, so that by some early epoch in galactic history most of the primordial gas had already been converted into stars. Wilson, on the other hand, on the basis of the intensity distribution of H and K spectral components in stars, finds that the stellar formation rate started slowly, reached a peak about the time of formation of 61 Cygni, and has been decreasing ever since. To cover this range of possibilities three forms for $\mathfrak{M}_g(t)$ have been chosen:

$$\mathfrak{M}_g(t) = \mathfrak{M}_g(0)e^{-at} \tag{7a}$$

$$\mathfrak{M}_g(t) = \mathfrak{M}_g(0)\ (1 + at)e^{-at} \tag{7b}$$

$$\mathfrak{M}_g(t) = \mathfrak{M}_g(0)\ (1 + at + 0.5\ a^2\ t^2)e^{-at} \tag{7c}$$

These functions are arranged in order of increasingly deferred periods of maximum stellar formation rates. Variation of the quantity a in the above relations allows a wide range of possibilities to be examined. As we shall see in Section VI, it is necessary to examine all the possibilities in equations (7) because each equation is attractive in some respects once a proper value for a is determined for it.

III. Stellar Masses, Lifetimes, and Luminosity Functions

We have chosen, as a representative and consistent set of relations between stellar mass, lifetime and initial luminosity function, those given by Limber (1960). Thus, the initial luminosity function for stars with absolute visual magnitudes brighter than +5 consists of an equally weighted mean of three sets of values; two by Sandage (1957) and one by van den Bergh (1957). One of those given by Sandage is derived by modifying the observed luminosity function of the solar neighborhood for the effects of evolution. The remaining two sets are based on the luminosity function of young galactic clusters. For stars fainter than +5 the observed solar neighborhood luminosity function is used, based on the assumption that this neighborhood constitutes a closed system whose faint stars have not yet evolved away from the main sequence. The values used are given in Table 1 and Figure 1, where $\Psi(M_V)$ is in arbitrary units.

It is estimated that the total mass of faint stars of magnitude greater than +20 not accounted for in the initial luminosity function is about five per cent of the total mass of stars which is actually present. In the notation of Section II this is equivalent to setting $F = 0.95$.

The stellar masses and lifetimes are derived by Limber from calculations of Schwarzschild and Härm (1958), Henyey, LeLevier and Levee (1959), and others, for stars on or near the main sequence. Not included in $\tau(M_v)$ are times spent during the later stages of evolution even though these times may represent perhaps as much as 30 per cent of the main sequence lifetime (Woolf 1962). Furthermore, the effects of differences in initial chemical compositions have not been taken into account in $\tau(M_v)$, nor has any attempt been made here to evaluate what these effects imply for this work. $\mathfrak{M}(M_v)$ and $\tau(M_v)$ are given in Table 1 and Figures 2 and 3, where $\tau(M_v)$ is in years and $\mathfrak{M}(M_v)$ is relative to one solar mass. $\tau(M_v)$ is not listed after an M_v of +6.0 since none of the calculations performed go beyond a galactic age of 30 billion years.

TABLE 1. Adopted Relations between Birth Rate Function, Mass, and Lifetime as Functions of Visual Magnitude.

M_V	$\Psi(M_V)$	$\mathfrak{M}(M_V)$	$\tau(M_V)$
-5	2.5	50	4.1×10^6
-4	6.4	27	6.8×10^6
-3	11.2	15.5	1.15×10^7
-2	20.0	9.4	2×10^7
-1	34.5	6.0	3.7×10^7
0	63.0	3.95	8×10^7
1	1.15×10^2	2.8	1.85×10^8
2	1.58 c 10^2	2.1	4.7×10^8
3	1.79×10^2	1.65	1.3×10^9
4	2.22×10^2	1.25	3.9×10^9
5	2.56×10^2	1.00	1.2×10^{10}
6	3.37×10^2	0.86	3×10^{10}
8	4.7×10^2	0.61	-
10	7.4×10^2	0.40	-
12	1.12×10^3	0.25	-
14	1.40×10^3	0.15	-
16	1.0×10^3	-	-
18	2.51×10^2	-	-
20	1.95	-	-

IV. Production of Heavy Elements

A determination of the abundances of the heavy elements demands that certain assumptions be made concerning the effects of advanced stellar evolution. We must calculate the mass fractions of the various constituents produced in these advanced stages as a function of stellar mass. The general prescriptions employed in the present work are illustrated in Figure 4 and will be elaborated in the following discussion.

Hayashi, Hoshi, and Sugimoto (1962) have carried models for stars of 0.7 $M_{\odot}$, 4 $M_{\odot}$, and 15.6 $M_{\odot}$ through the phase of carbon burning. The depletion of the various nuclear fuels results in models of inhomogeneous chemical composition. Although our interest is with the advanced evolutionary stages, it is appropriate to survey briefly the histories of these stars.

A star of 15.6 $M_{\odot}$, at the onset of hydrogen burning, consists of a convective core and a radiative envelope. Hydrogen burning in the core is accompanied by a decrease in the mass of the core and the growth of a radiative region of varying hydrogen composition between the core and the envelope. During the final stages of hydrogen burning in the core, hydrogen begins to burn in a shell source surrounding the core.

The exhaustion of the hydrogen fuel in the core is followed by the contraction of the core, the temperature increasing until the helium in the core ignites. In this initial stage of helium burning the depletion of hydrogen in the shell source results in the growth of the helium core and a contraction of the hydrogen envelope. The structure of the star at this stage consists of a convective helium burning core, an intermediate radiative helium zone, and a radiative hydrogen envelope. Hydrogen burning continues in the shell source through the final stages of helium burning.

The depletion of helium in the core results finally in the contraction of the core and the onset of carbon burning. In the early phases of carbon burning the star consists of a convective carbon burning core, a radiative carbon region, a radiative helium region, and a convective hydrogen envelope. The hydrogen shell source is no longer active, but helium burning proceeds in a shell source at the base of the radiative helium zone.

For the purpose of our calculations we have assumed that the compositional structure of a star in the final phase of carbon burning corresponds to the structure at end of its life.

As the central temperature increases still further, neutrino pair emission should speed up the subsequent stages of stellar evolution so much, that very little further change in the composition of the outer parts of the star is likely. For a star of 15.6 $M_\odot$ in the final phase of carbon burning (Hayashi et al, 1962) helium has been destroyed out to a mass fraction q_2 = 0.22 of the star, and hydrogen out to a mass fraction q_1 = 0.272. The helium burning shell source, which is relevant to our discussion of the radioactivities, is situated at q_2. In our treatment we are not concerned with the degree of depletion of carbon in the core.

Although the details of the early evolution of less massive stars, $\mathfrak{M} < 4\ M_\odot$, differ somewhat from our previous discussion, the exhaustion of hydrogen in the core finds them similarly composed of a helium core, a hydrogen shell source, and a radiative hydrogen envelope. A star of 0.7 $M_\odot$ will undergo a helium flash due to the condition of electron degeneracy in the core.

A star of 4 $M_\odot$ will not experience a helium flash, but will proceed as did the 15.6 $M_\odot$ star into the helium burning stage. As helium depletion continues in the core the hydrogen shell source becomes inactive. Election degeneracy increases with the growth of the core; thus the exhaustion of helium in the core is followed by a carbon flash. Carbon burning proceeds in the core and through these phases the structure of a star of 4 $M_{\odot}$ is similar to that found for a star of 15.6 $M_{\odot}$. The final phase of carbon burning finds helium exhausted out to a mass fraction $q_2 = 0.258$, and hydrogen to $q_1 = 0.288$. There is a helium burning shell source at q_2, the base of the radiative helium zone. Again the subsequent lifetime is likely to be very short with little change in the outer regions.

For a star of 0.7 $M_{\odot}$ the hydrogen shell source is also inactive in the early stages of helium burning. A degenerate core will form as in the case of the 4 $M_{\odot}$ star. In this instance, however, the temperature never increases to the point at which a carbon flash can occur. The important development for our consideration is the reactivation of the hydrogen burning shell source. This is found to take place in the later phases of helium burning as the helium burning core approaches the hydrogen envelope (Hayashi, 1964). Under these conditions hydrogen depletion will take place to a large extent. Later the helium shell source becomes inactive.

Table 2 summarizes the results of Hayashi, Hoshi, and Sugimoto (1962) for the mass fractions within which hydrogen and helium have been exhausted, q_1 and q_2, as a function of stellar mass.

Table 2

Mass Fractions in which Hydrogen and Helium have been Exhausted

Mass	q_1	q_2
0.7 $M_\odot$	.76	.68
4	.288	.258
15.6	.272	.22

These results correspond to the conditions prevailing in the final phases of carbon burning for the more massive stars. For purposes of computation we have determined quadratic fits to these numbers for the total mass interior to the hydrogen and helium burning shells as a function of stellar mass:

$$q_1 \mathfrak{M} = 0.00527 \mathfrak{M}^2 + 0.1623 \mathfrak{M} + 0.413 \qquad (8)$$

$$q_2 \mathfrak{M} = 0.00264 \mathfrak{M}^2 + 0.1552 \mathfrak{M} + 0.365 \qquad (9)$$

In these and in the ensuing expressions all masses will be in solar mass units.

The value of q_1 quoted above for a star of 0.7 $M_\odot$ does not include the influence of the reactivation of the hydrogen burning shell source. In an attempt to incorporate this effect for the small mass stars we have assumed that hydrogen is completely exhausted ($q_1 = 1$) in all stars of mass less than $M_\odot$. This assumption projects forward Hayashi's calculations and assumes that no mass loss occurs until the very latest stage of the star's evolution. The region of hydrogen exhaustion for stars of mass greater than $M_\odot$ is taken to be equal to $M_\odot$ until $q_1 \mathfrak{M}$ defined by equation (8) becomes equal to $M_\odot$. It is this general prescription which defines the hydrogen burning shell exhibited in Figure 4.

The stars in our model are divided into two distinct classes: small mass stars which evolve to white dwarfs and massive stars which are assumed to become supernovas and to leave behind an imploded remnant which, if stable, might become a neutron star. Some assumptions must be made concerning the nature of these evolutionary remnants. In our calculations we have assumed that all stars of mass greater than 4 $M_\odot$ will become supernovas. The mass of the imploded remnant is taken to be 0.175 of the mass of the star: a star of 4 $M_\odot$ then leaves a remnant of 0.7 $M_\odot$. The end point of evolution for stars of mass less than 4 $M_\odot$ is assumed to be a white dwarf. The white dwarf mass is taken to be 0.7 $M_\odot$ for all stars satisfying $0.7 \leq q_2 \mathfrak{M}$ In order to maximize helium ejection to the interstellar medium, for those stars for which the mass interior to the helium burning shell is less than 0.7 $M_\odot$, the mass of the white dwarf remnant is taken to be equal to q_2 M: stars in this range contribute no heavy elements to the interstellar gas. These general prescriptions define the helium burning shell and the appropriate remnants as exhibited in Figure 4.

It is important to note the problems associated with the choice of these various prescriptions. In general Hayashi's results for the more massive stars, $\mathfrak{M} \geq 4$ $M_\odot$, reveal that little helium will be produced. For a 15.6 $M_\odot$ star the mass fraction of the helium zone, $q_1 - q_2$, is .052 while for the 4 $M_\odot$ case it is only .03. Furthermore, these mass fractions are small compared to the mass fraction q_2 of heavy elements in the same stars. Thus even if the past rate of formation of these massive stars has been much greater than the present rate it would be difficult to produce large amounts of helium either absolutely or relative to heavy elements.

If the galaxy was initially composed of pure hydrogen, it seems apparent that the evolution of small mass stars must account for the helium content of the interstellar gas. The reactivation of the hydrogen shell source in the late stages of helium burning for stars of mass $\mathfrak{M} < M_\odot$ might well provide the necessary helium. However, the long lifetimes associated with these stars imply high galactic ages.

The size of the evolutionary remnants is the determining factor in the mass of heavy elements produced in these models. Greenstein (1958) has given an average value of 0.55 $M_\odot$ for a white dwarf remnant. Our value of 0.7 $M_\odot$ is chosen to fit smoothly to the mass of the imploded remnant at 4 $M_\odot$ although there is no physical necessity for continuity. For those stars with $0.7 > q_2 \mathfrak{M}$ the white dwarf remnant is assumed to have mass $q_2 \mathfrak{M}$. By this prescription we decrease the average white dwarf mass to be comparable to Greenstein's determination and enhance the production of helium. The choice of the mass of the imploded remnant as 0.175 M is governed by the need to produce reasonable heavy element abundances, although it is difficult to change their production by more than a factor two by using other remnant masses. It is interesting to note that our prescriptions result in the production of approximately equal masses of heavy elements from the two classes of stars.

We have further assumed that secondary heavy elements are produced in supernovas. The radioactivities discussed in Section V are assumed to be formed in this manner. In fact, all these nuclei except K^{40} are produced by neutron capture on a fast time scale, defined by the condition that neutron capture lifetimes are short compared to beta decay half-lives along the capture path. It is believed that this situation will be realized in supernova explosions (Cameron, 1962).

The implosion of the stellar core, in this view, results in the formation of a shock wave which will propagate outwards through the star. The important conclusion for our present considerations is that when this shock wave passes through a region in which helium still exists, a number of (α,n) reactions are initiated resulting in a strong flux of neutrons. This neutron flux is required to produce the fast time scale capture products in which we are interested. But the production of heavy elements requires the previous operation of neutron capture on a slow time scale in this helium region, thus restricting us to the helium-burning shell.

In keeping with this picture we have assumed that the production of the secondary heavy elements is proportional to the mass of the helium burning shell source in Hayashi's models. Again following Hayashi's (1962) models for the phase of carbon burning, the radius of the helium burning shell source for a star of $4\ M_{\odot}$ is $r_2 = 2.59 \times 10^{9}$ cm. ($q_2 = .258$). The temperature and density at this point are given by $T = 1.66 \times 10^{8}\ ^{\circ}K$ for the helium mass fraction and

$\rho = 2.4 \times 10^3$ gm/cm^3. If we take for the helium mass fraction $Y = 1$, the energy generation for this region from the triple alpha process is $\epsilon = 3.75 \times 10^6$ erg/gm.sec. From the value of the helium burning luminosity, $L = 1.28 \times 10^{37}$ erg/sec., we can determine the mass of the helium burning shell source, $L/\epsilon = 3.4 \times 10^{30}$ gms. Similarly for a star of 15.6 $M_\odot$ we have $r_2 = 5.4 \times 10^9$ cm., $T = 2.14 \times 10^8$ °K, $\rho = 5.01 \times 10^2$ gm/cm^3, $\epsilon = 3.21 \times 10^7$ erg/gm. sec., and $L = 3.44 \times 10^{38}$ erg/sec. The mass of the helium shell source in this case is then 1.07 1.07×10^{31} gms. Fitting these two points, we find for the mass of the helium burning shell source for a star of mass M

$$\mathfrak{M}_{He} \propto .063\ \mathfrak{M} + .088 \qquad (10)$$

The exact factors of proportionality are not required here, as we will be interested only in ratios of the secondary heavy nuclei. This eliminates the error in our choice of $Y = 1$ in the previous calculations.

Having defined the structure of our two classes of stars in the final stages of evolution, the abundances of the various constituents of the interstellar gas can be followed directly. A star formed at time t_i is formed with mass fractions $X(t_i)$, $Y(t_i)$, and $Z(t_i)$ of hydrogen, helium, and heavy elements. In our calculations the masses of heavy elements from the two classes of stars are followed individually, Z being the sum of these two contributions. A star of mass $\mathfrak{M}$ has an associated lifetime $\tau(\mathfrak{M})$ determining that time $t_1 + \tau$ at which it will evolve and enrich the interstellar gas. The total mass released at the end of the life is simply $\mathfrak{M}$, less the mass of the appropriate remnant. The mass fractions of hydrogen, helium, and heavy elements released are determined by the prescriptions established above.

It is further assumed that the hydrogen region, mass $\mathfrak{M} - q_1\mathfrak{M}$, is not composed purely of hydrogen but rather it contains the abundances at formation, $X(t_1)$, $Y(t_1)$, and $Z(t_1)$. Similarly, the helium region, mass $q_1 \mathfrak{M} - q_2 \mathfrak{M}$, contains a mass fraction $Z(t_1)$ of heavy elements.

At time $t_1 + \tau$ a whole distribution of stars will evolve of varying lifetime, hence varying initial composition. The total mass of the various constituents released by these stars is assumed to be mixed with the interstellar gas. Stars born at this time are formed with the updated mass fractions of hydrogen, helium, and heavy elements.

V. Treatment of Radioactivities

A knowledge of the primordial abundances of various radioactive nuclei and their decay products becomes important in treating the problems of cosmochronology and the problem of the general history of the solar system. In our treatment of galactic evolution we have traced the abundances of five such nuclei: these isotopes and their decay constants

$$\lambda = \frac{0.693}{T_{1/2}} \text{ years}^{-1} \tag{11}$$

are given in the following table.

Table 3

Radioactive Isotopes and their Decay Constants

Isotope	Decay Constant(λ)
Th^{232}	0.498×10^{-10}
U^{238}	0.154×10^{-9}
K^{40}	0.555×10^{-9}
U^{235}	0.976×10^{-9}
I^{129}	0.406×10^{-7}

The decay of the radioactive nuclei was handled in the usual manner. The number of nuclei remaining at time t, N(t), is related to the number present initially, N_0, by:

$$N(t) = N_0 \exp(-\lambda t) \tag{12}$$

In this formula it is assumed that all of the radioactive mother nuclei are formed at the same time interval. If we assume, rather, that the radioactivity is formed at a constant rate over some interval, T, then at the end of this interval the ratio of the abundance of the radioactivity remaining to the total amount formed in the interval is .

$$\frac{N}{N_{tot}} = \frac{1}{\lambda T}\left[1 - \exp(-\lambda T)\right] \tag{13}$$

This formula was employed, in particular, for the case of I^{129}. All of the other radioactivities satisfy the condition that their half-lives be long compared to the chosen intervals of time integration.

If we are to be able to interpret correctly our results for the ratios of these activities, we must consider in some detail the accepted values for these ratios and their corresponding uncertainties. The two ratios with which we shall be most concerned are U^{235} / U^{238} and Th^{232} / U^{238}. Following Cameron (1962) the present ratio of U^{235} to U^{238} abundances is 0.00723. The differential mean life of U^{235} relative to U^{238} is 1.22×10^{9} years (Fowler and Hoyle, 1960). The age of the solar system is taken to be 4.55×10^{9} years (Patterson, 1956). This age refers to the chemical isolation of lead from uranium.

This treatment assumes furthermore that the earth and meteorites were formed simultaneously with the sun. The primordial ratio of U^{235} / U^{238} is thus found to be 0.00723 exp (4.55/1.22) = 0.301, with an associated error of approximately $\pm 10\%$. The differential mean life of Th^{232} relative to U^{238} is 9.63×10^9 years. The value employed by Cameron (1962) for the present ratio of Th^{232} to U^{238} was 3.8 ± 0.3 (Fowler and Hoyle, 1960), resulting in a value for the primordial ratio of 2.37 ± 0.19. Lovering and Morgan (1964) find a value 4.27 for this ratio from a study of olivine - pigeonite and ordinary chondrites in which the uranium and thorium abundances were determined simultaneously. Employing this value for the present ratio, we find the primordial ratio of Th^{232} to U^{238} to be 4.27 exp (-4.55/9.63) = 2.66, with an error of about $\pm$ 20%.

These secondary heavy elements are assumed to have been made by a process of neutron capture on a fast time scale. The production ratio of U^{235} to U^{238} we shall adopt is 1.45 $\pm$ 20% (Cameron, 1962). The production ratio of Th^{232} to U^{238}, if all progenitors are formed with equal abundance, is 1.85. An increase of the abundance slope of 10% per mass number yields a ratio 2.02; a similar decrease of 10% per mass number results in a ratio 2.24. We shall take 2.10 as a probable upper limit on the production ratio. If there were an abundance maximum on the fast capture path near mass number 240, the progenitors of U^{238} would be favored, and the production ratio would fall to approximately 1.65 (Fowler and Hoyle, 1960). Taking this value as a probable lower limit, we find for the production ratio of Th^{232} to U^{238}, $1.85 \pm {}^{.25}_{.20}$.

VI. Results and Discussion

The results of our investigation are summarized in Table 4. Each of the three prescriptions employed for the rate of star formation (equation 7) has been examined for values of 'a' corresponding to the condition that 5% of the original gas remains at 10, 15, 20, and 25 billion years, respectively. In this table τ_s denotes the time of formation of the sun, determined by that time at which the helium abundance of the interstellar gas is Y = 0.24 (Gaustad, 1964). The heavy element content of the interstellar gas and the ratios of the radioactivities as obtained by our models for τ_s are also tabulated.

If we assume that the solar system is 4.55×10^9 years old (Patterson, 1956), we can then predict, from our models, the conditions for the present day galaxy. For $\tau_s + 4.55$ we have tabulated the mass fractions Y and Z of helium and heavy elements in the interstellar gas. The fractional mass of the galaxy in the form of evolutionary remnants and in stars of lifetime greater than 30 billion years is also presented.

The compositional histories of the galaxy for various of these models are illustrated in Figures 5a, 5b, and 5c. For these same models the ratios U^{235} / U^{238} and Th^{232} / U^{238} are followed in time in Figures 6a, 6b, and 6c. The probable

TABLE 4. The Results of these Calculations are Tabulated for a Variety of Prescriptions.

Prescription	a	τ_s	$\frac{U^{235}}{U^{238}}$	$\frac{Th^{232}}{U^{238}}$	K^{40}/Stable	I^{129}/Stable	Z	τ_s+4.5	Y	Z	Fractional Mass In Imploded Remnants	Fractional Mass In White Dwarfs	Fractional Mass $\tau > 30$
$M_g=M_o e^{-at}$	2.996×10^{-10}	9.7	.318	2.90	1.80×10^{-1}	2.83×10^{-3}	.0217	14.2	.52	.0274	.012	.108	0.609
	1.997×10^{-10}	13.9	.288	3.29	1.33×10^{-1}	1.99×10^{-3}	.0204	18.4	.41	.0252	.111	.111	0.602
	1.498×10^{-10}	17.7	.274	3.60	1.07×10^{-1}	1.57×10^{-3}	.0197	22.2	.36	.0236	.010	.114	0.598
	1.198×10^{-10}	21.5	.265	3.87	9.00×10^{-2}	1.30×10^{-3}	.0191	26.0	.34	.0225	.0098	.117	0.594
$M_g=M_o(1+at)e^{-at}$	4.74×10^{-10}	10.2	.378	2.65	2.39×10^{-1}	4.04×10^{-3}	.0216	14.7	.62	.0272	.0091	.105	0.606
	3.16×10^{-10}	14.7	.328	2.98	1.75×10^{-1}	2.75×10^{-3}	.0205	19.2	.48	.0263	.0085	.110	0.604
	2.37×10^{-10}	18.9	.302	3.25	1.39×10^{-1}	2.11×10^{-3}	.0199	23.4	.40	.0247	.0083	.113	0.600
	1.90×10^{-10}	23.1	.287	3.50	1.15×10^{-1}	1.71×10^{-3}	.0193	27.6	.36	.0235	.0081	.116	0.597
$M_g=M_o(1+at+0.5a^2t^2)e^{-at}$	6.30×10^{-10}	10.4	.410	2.55	2.72×10^{-1}	4.83×10^{-3}	.0218	14.9	.70	.0263	.0083	.104	0.607
	4.20×10^{-10}	14.9	.351	2.83	2.00×10^{-1}	3.25×10^{-3}	.0208	19.4	.52	.0270	.0081	.108	0.605
	3.15×10^{-10}	19.4	.319	3.10	1.58×10^{-1}	2.45×10^{-3}	.0202	23.9	.44	.0257	.0081	.112	0.603
	2.52×10^{-10}	23.7	.301	3.33	1.31×10^{-1}	1.98×10^{-3}	.0197	28.2	.38	.0245	.0080	.115	0.600
Adopted Values			$.301 \pm .607$	$2.66^{+.62}_{-.56}$			.021		.35				

error limits indicated on these graphs include the uncertainties both in the production ratios and in the accepted values for the primordial abundance ratios. The ratios of the I^{129} and K^{40} abundances to that of a stable nuclei formed by the same prescription are plotted in Figures 7 and 8 for the time interval $\tau_s - 5$ to $\tau_s + 5$. These results are included because of their interest for problems of nucleosynthesis and cosmochronology, but we shall not attempt an interpretation in this paper.

In this investigation we have sought to determine the extent to which the various prescriptions for the rate of star formation result in reasonable values for the composition of the interstellar gas. Assuming that the helium content of the sun is 24% by mass, we can determine the time of formation of the sun from the interstellar gas. The mass fraction of heavy elements at this time should also approximate that observed in the sun, .021 (Aller, 1961). The results of all our models agree reasonably well with this value.

It is evident that we can gain little insight from an examination of the ratios of radioactivities. The results of our models for U^{235} / U^{238} and Th^{232} / U^{238} agree with the accepted solar system primordial values within the limits of probable error over a wide range of prescriptions. Furthermore, as is apparent in Figure 6, for a given prescription these ratios lie within the region of uncertainty over a broad range of galactic ages.

The helium content of the gas at the present time can be inferred from a study of the compositions of O and B stars. A reasonable value for $Y(\tau_s + 4.5)$ is .35 (Aller, 1961). The values of the present helium content of the gas in our models vary quite noticeably with the prescription for the rate of star formation. The need to satisfy $Y = .35$ at the present time leads us toward high galactic ages.

This study was undertaken to determine whether the high helium content of the sun and the O and B stars might result from stellar evolution starting with a pure hydrogen galaxy. So many uncertain assumptions must be made that such a study cannot now determine whether the pure hydrogen assumption is allowed or not. But we consider it to be significant that we have produced models of galactic history, based on what we believe to be reasonable assumptions, in which satisfactory values of the helium contents of the sun and of the O and B stars can be produced starting with a pure hydrogen galaxy. A large galactic age ($\gtrsim 2 \times 10^{10}$ years) is required for such models. This age may be greatly extended if star formation is turned on gradually. In all of our models the heavy element content of the interstellar medium rapidly rises and then changes little with time, whereas the helium content rises much more slowly. This behavior of the heavy elements is qualitatively confirmed by abundance analyses of stars.

In our models we also find the ratios of principal long-lived radioactivities to be approximately those observed for the solar system, at the time of its formation, over a large part of galactic history, including the time that the interstellar medium acquires a helium content typical of the sun. Hence we conclude that our models are consistent with requirements on the radioactivities, but we also conclude that no useful time can be derived for the onset of nucleosynthesis in the galaxy.

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Figure Captions

Figure 1. The adopted initial luminosity function is presented as a function of visual magnitude.

Figure 2. The mass of a star is plotted as a function of visual magnitude.

Figure 3. The lifetime of a star on the main sequence is presented as a function of visual magnitude.

Figure 4. The compositional structure of a star as a function of mass in the final stage of evolution.

Figure 5. The compositional histories of the galaxy are traced for the indicated prescriptions for the gas content function. The time of formation of the sun corresponds to the condition that the helium mass fraction $Y = 0.24$.

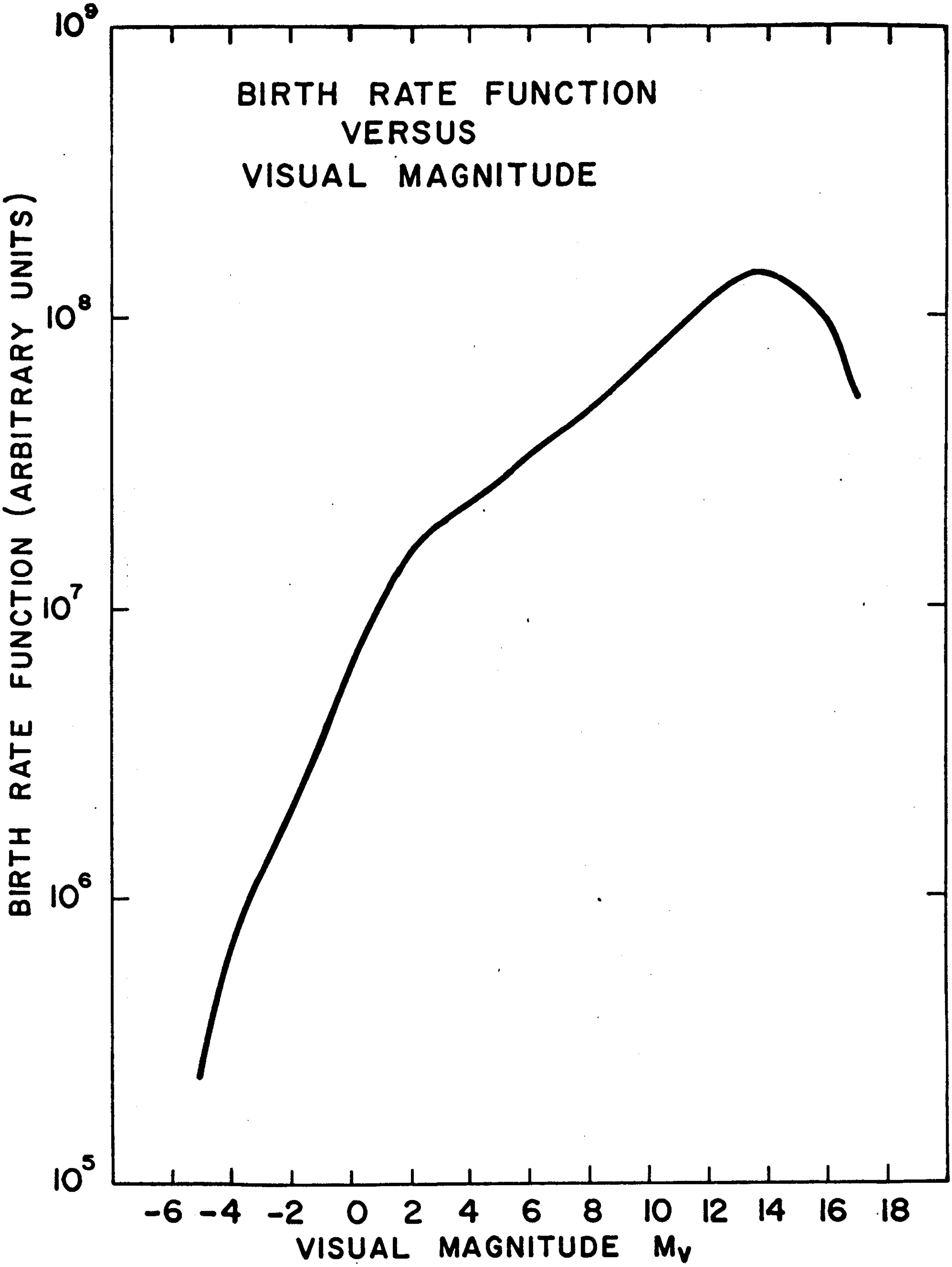
Figure 6. The nuclide ratios U^{235} / U^{238} and Th^{232} / U^{238} are illustrated as a function of galactic age for the indicated gas content prescriptions.

Figure 7. The calculated ratio of the I^{129} abundance to that of a stable nucleus formed by the same prescription is plotted for a 10 billion year interval centered on the time of formation of the sun for a variety of models. (1.) $\mathfrak{M}_g = \mathfrak{M}_o e^{-at}$, $a = 2.996 \times 10^{-10}$ (2.) $\mathfrak{M}_g = \mathfrak{M}_o e^{-at}$, $a = 1.997 \times 10^{-10}$ (3.) $\mathfrak{M}_g = \mathfrak{M}_o e^{-at}$, $a = 1.498 \times 10^{-10}$

(4.) $m_g - m_o \; e^{-at}$, $a = 1.198 \times 10^{-10}$ (5.) $m_g = m_o \times$ $\times (1 + at)e^{-at}$, $a = 1.90 \times 10^{-10}$ (6.) $m_g = m_o \, (1 + at + 0.5a^2t^2)e^{-at}$, $a = 2.52 \times 10^{-10}$.

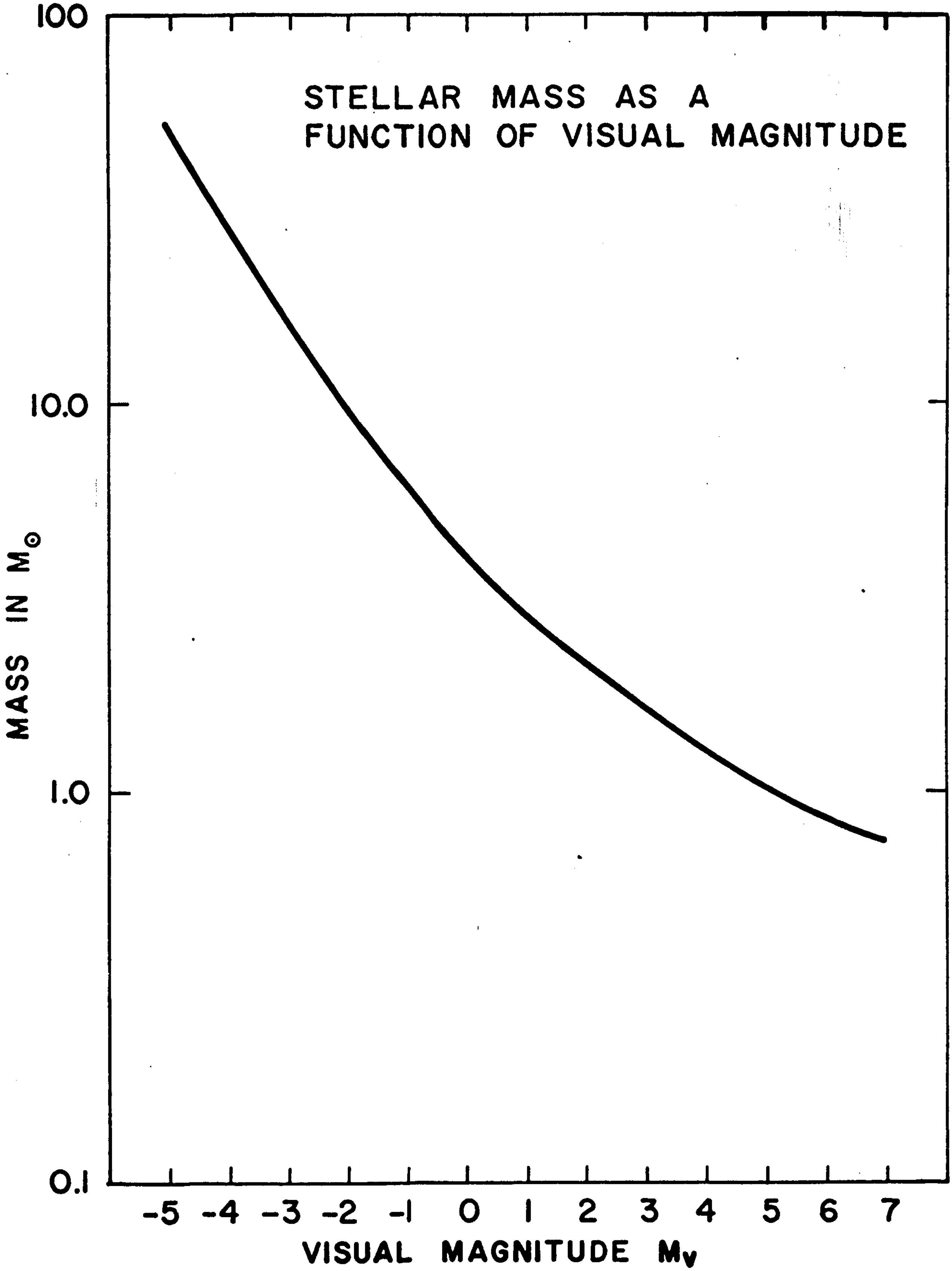
Figure 8. The calculated ratio of the K^{40} abundance to that of a stable nucleus formed by the same prescription is plotted for a 10 billion year interval centered on the time of formation of the sun for a variety of models. (1.) $m_g = m_o \; e^{-at}$, $a = 2.996 \times 10^{-10}$ (2.) $m_g = m_o \; e^{-at}$, $a = 1.997 \times 10^{-10}$ (3.) $m_g = m_o \; e^{-at}$, $a = 1.498 \times 10^{-10}$ (4.) $m_g = m_o \; e^{-at}$, $a = 1.198 \times 10^{-10}$ (5.) $m_g = m_o \times$ $\times (1 + at)e^{-at}$, $a = 1.90 \times 10^{-10}$ (6.) $m_g = m_o \, (1 + at + 0.5a^2t^2)e^{-at}$, $a = 2.52 \times 10^{-10}$.

FIGURE 1



BIRTH RATE FUNCTION
VERSUS
VISUAL MAGNITUDE
BIRTH RATE FUNCTION (ARBITRARY UNITS)
10^9
10^8
10^7
10^6
10^5
-6 -4 -2 0 2 4 6 8 10 12 14 16 18
VISUAL MAGNITUDE Mv

FIGURE 2



STELLAR MASS AS A
FUNCTION OF VISUAL MAGNITUDE
100
10.0
1.0
0.1
MASS IN $M_\odot$
-5 -4 -3 -2 -1 0 1 2 3 4 5 6 7
VISUAL MAGNITUDE M_V

FIGURE 3

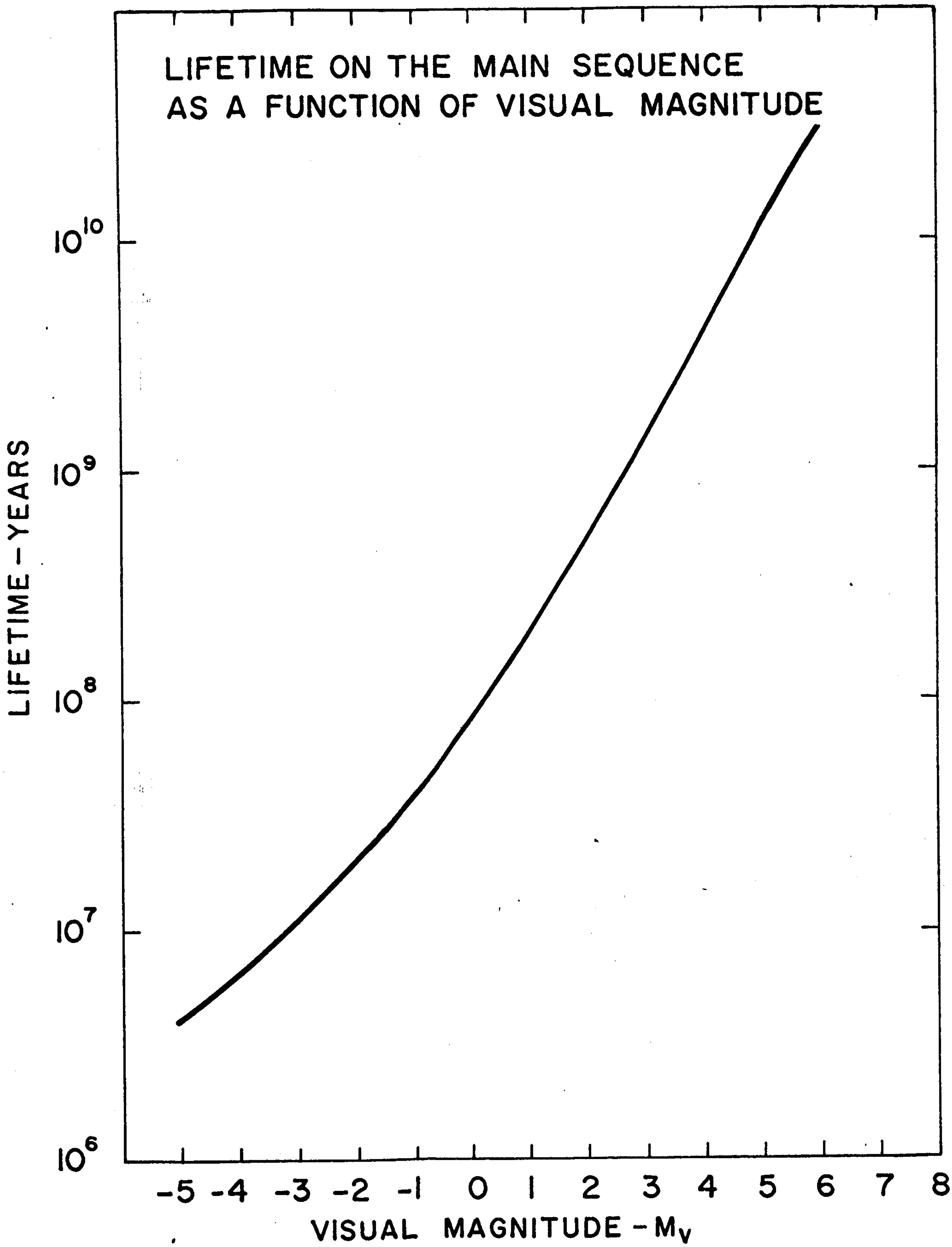
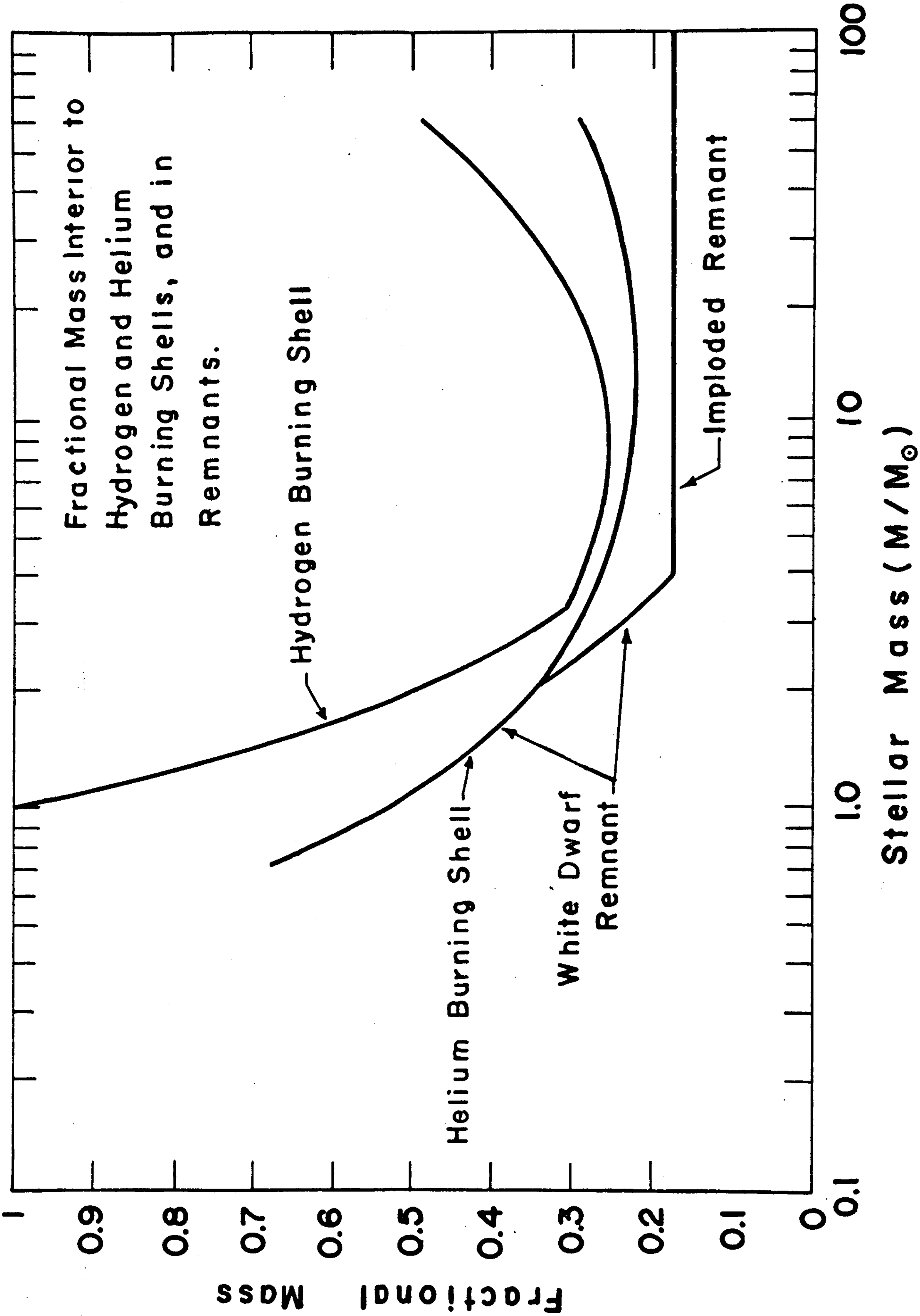
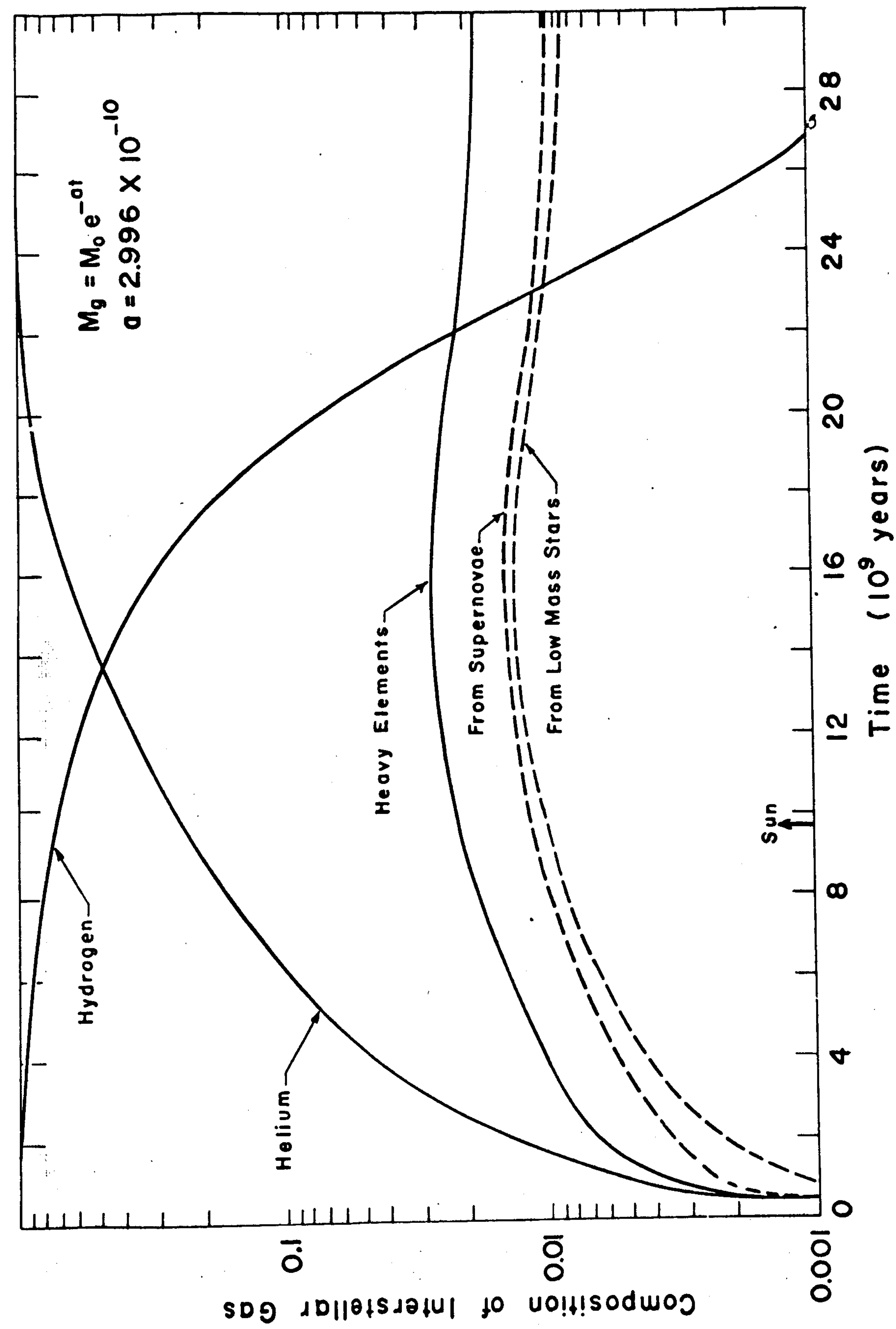

LIFETIME ON THE MAIN SEQUENCE
AS A FUNCTION OF VISUAL MAGNITUDE
10^{10}
10^9
10^8
10^7
10^6
LIFETIME - YEARS
-5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8
VISUAL MAGNITUDE - M_V

FIGURE 4



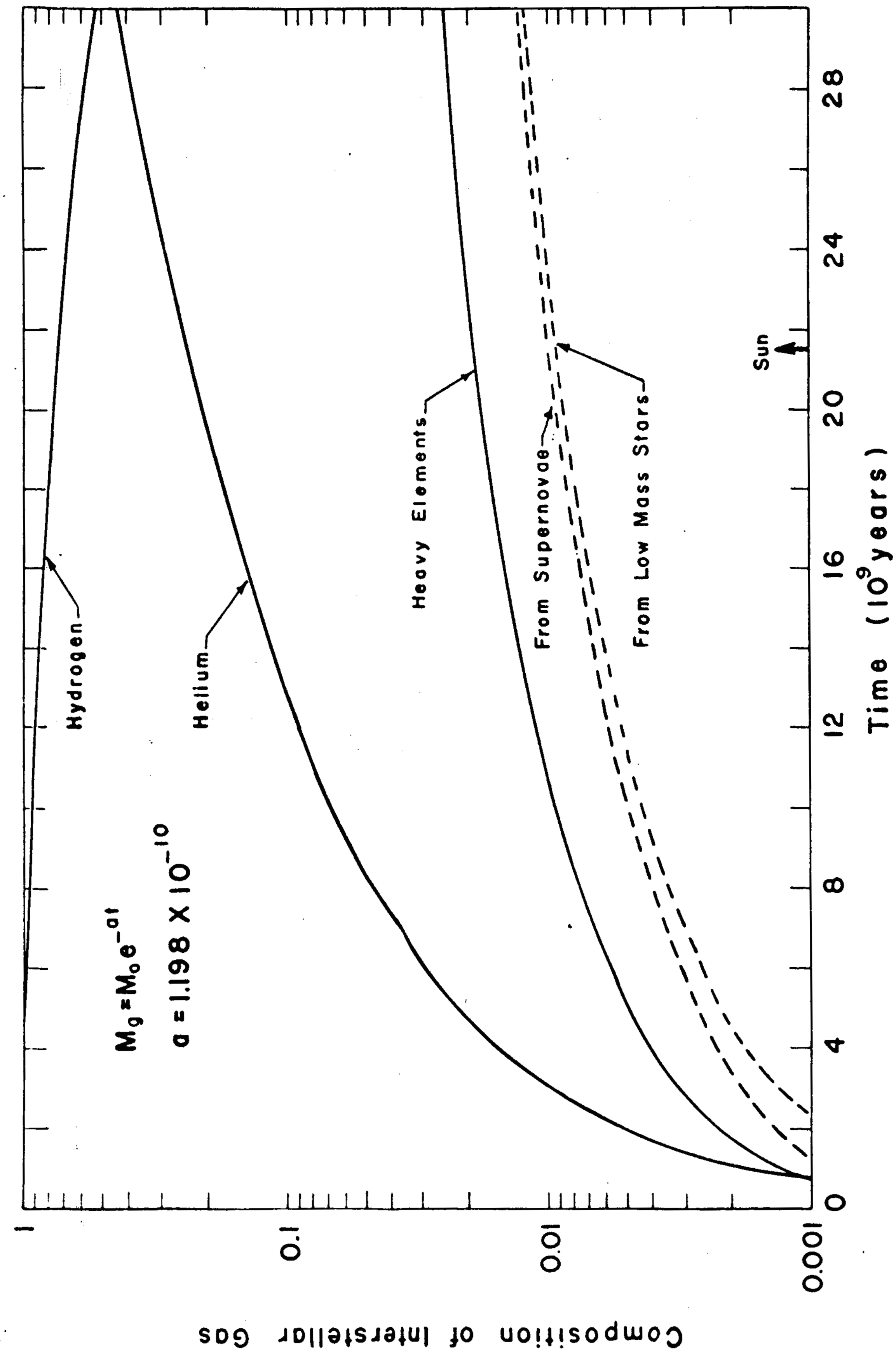
Fractional Mass Interior to Hydrogen and Helium Burning Shells, and in Remnants.
Hydrogen Burning Shell
Helium Burning Shell
White Dwarf Remnant
Imploded Remnant
Stellar Mass (M/M$_\odot$)
Fractional Mass
0.1
1.0
10
100
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

FIGURE 5a



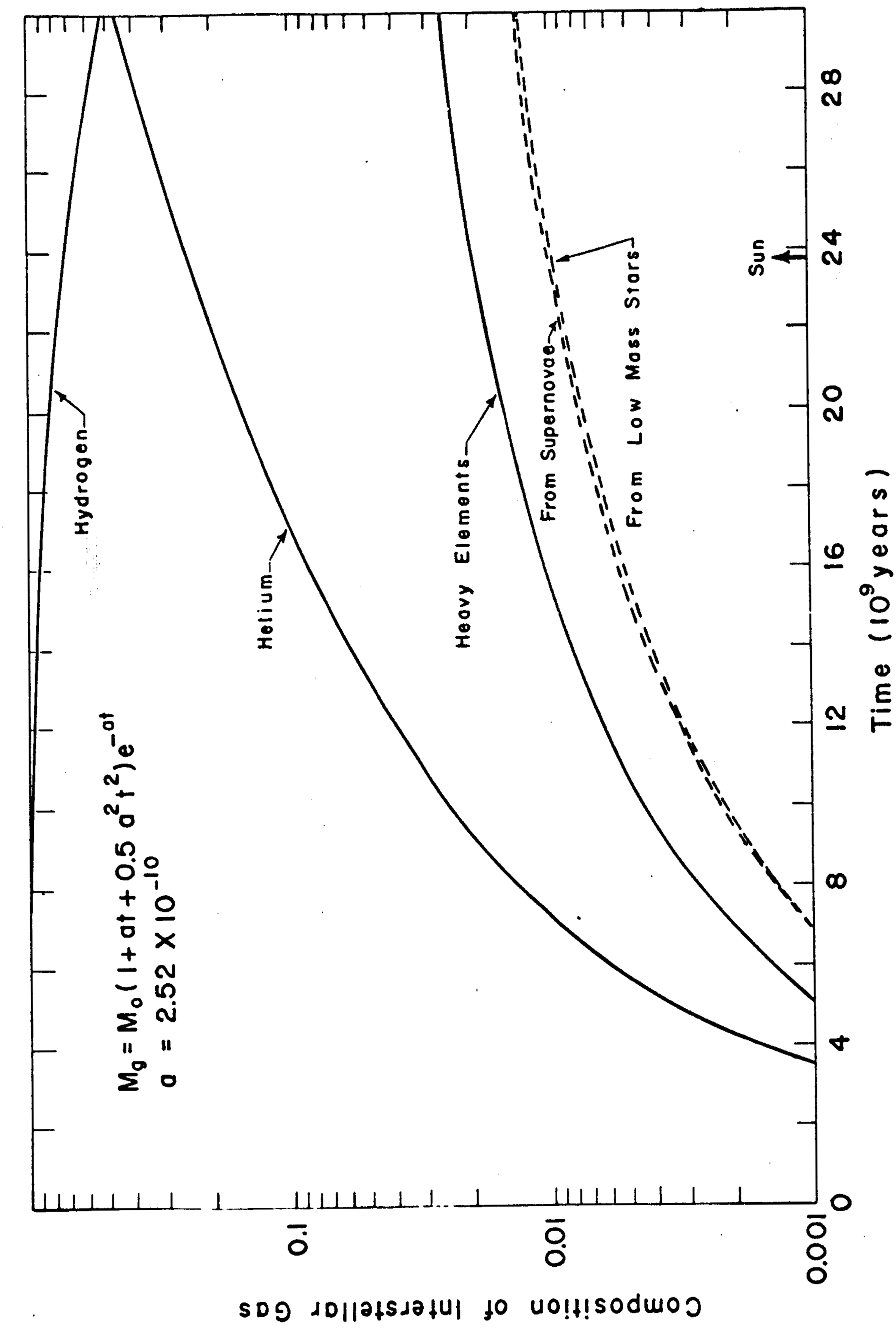
$M_g = M_0 e^{-at}$
$a = 2.996 \times 10^{-10}$
Hydrogen
Helium
Heavy Elements
From Supernovae
From Low Mass Stars
Sun
0
4
8
12
16
20
24
28
Time (10^9 years)
0.1
0.01
0.001
Composition of Interstellar Gas

FIGURE 5b



Composition of Interstellar Gas
Time (10^9 years)
$M_g = M_o e^{-at}$
$a = 1.198 \times 10^{-10}$
Hydrogen
Helium
Heavy Elements
From Supernovae
From Low Mass Stars
Sun
1
0.1
0.01
0.001
0
4
8
12
16
20
24
28

FIGURE 5c



$M_g = M_o(1 + at + 0.5\,a^2t^2)e^{-at}$
$a = 2.52 \times 10^{-10}$
Hydrogen
Helium
Heavy Elements
From Supernovae
From Low Mass Stars
Sun
0
4
8
12
16
20
24
28
Time (10^9 years)
0.1
0.01
0.001
Composition of Interstellar Gas

FIGURE 6a

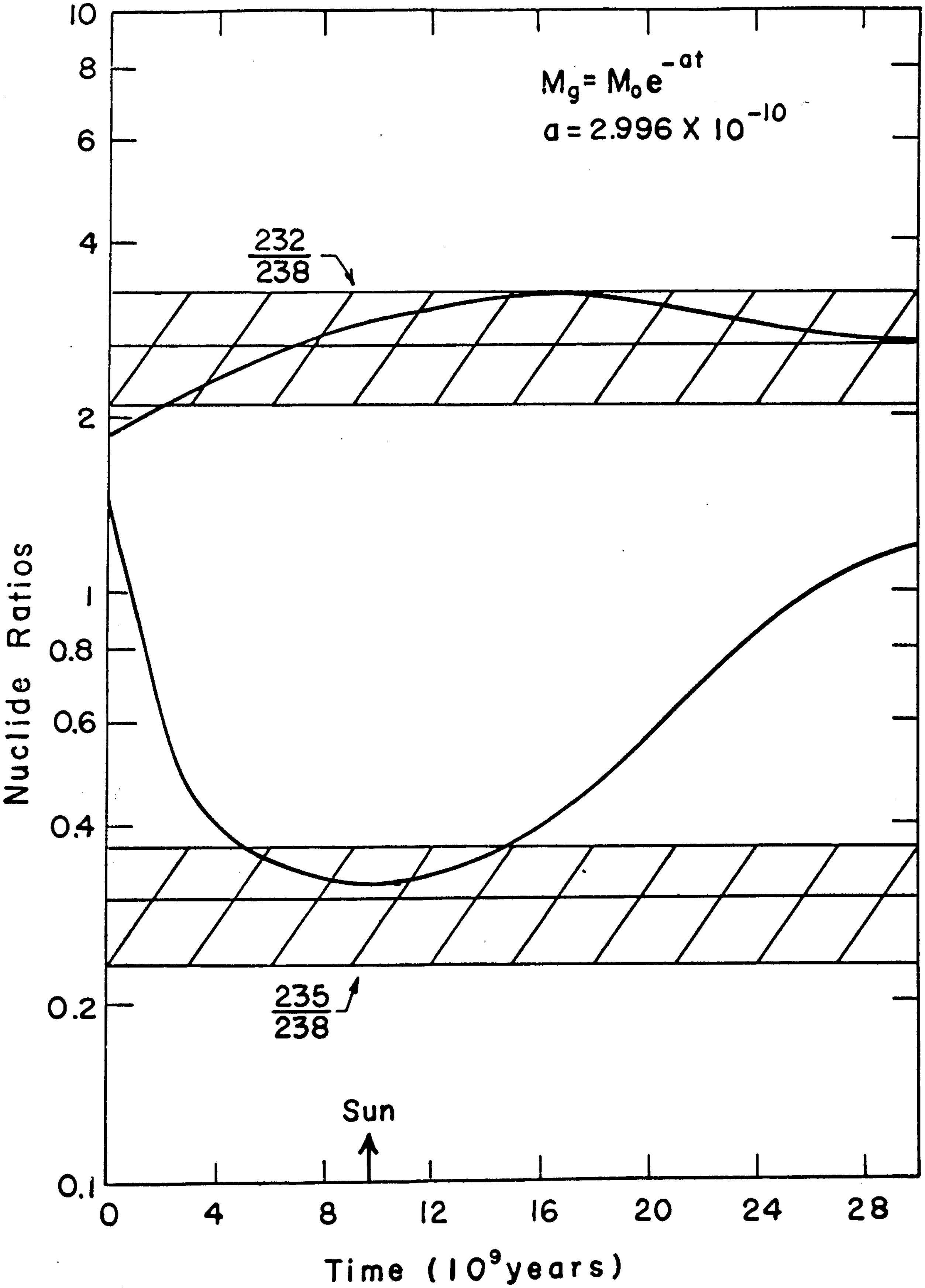
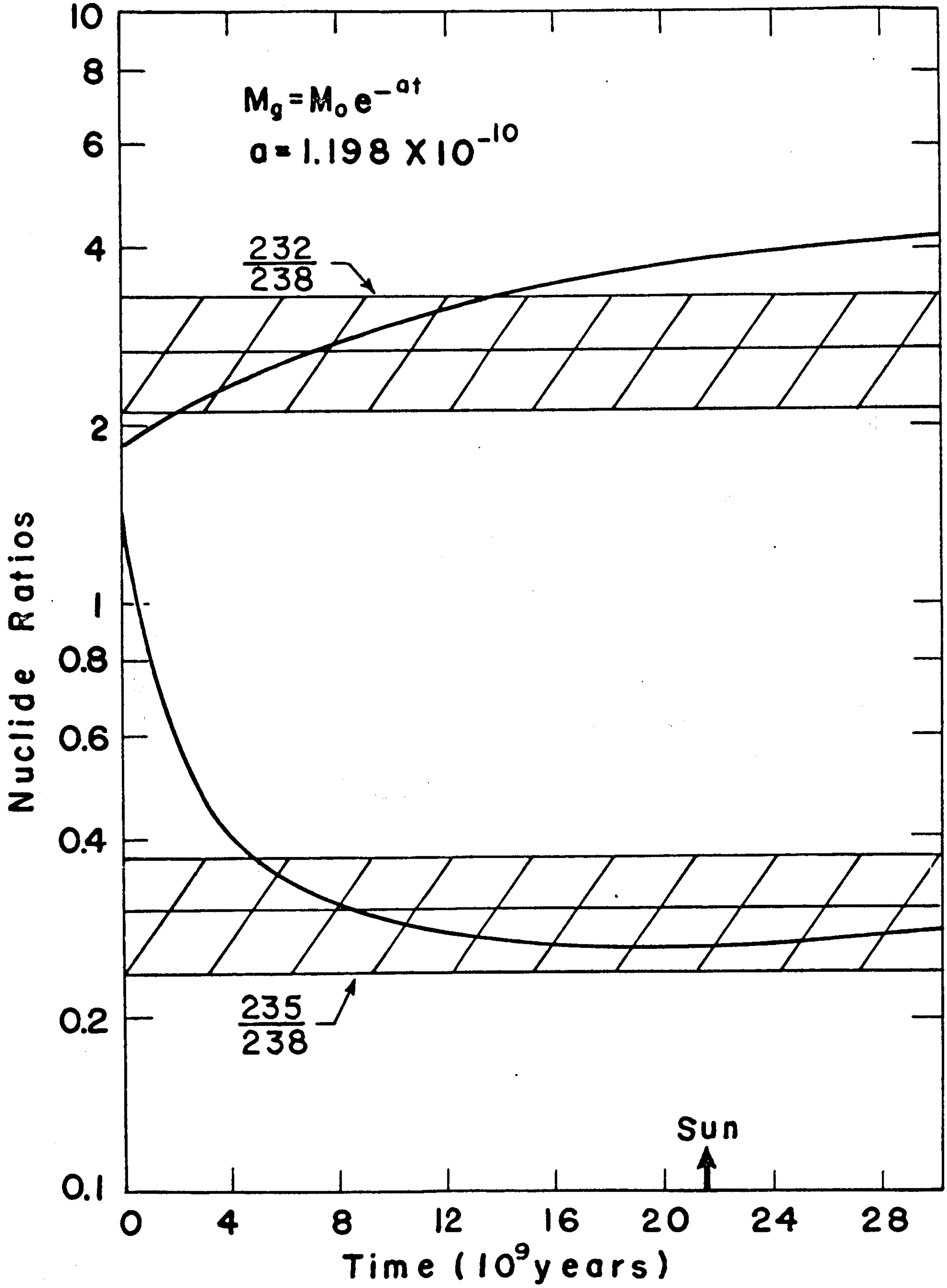
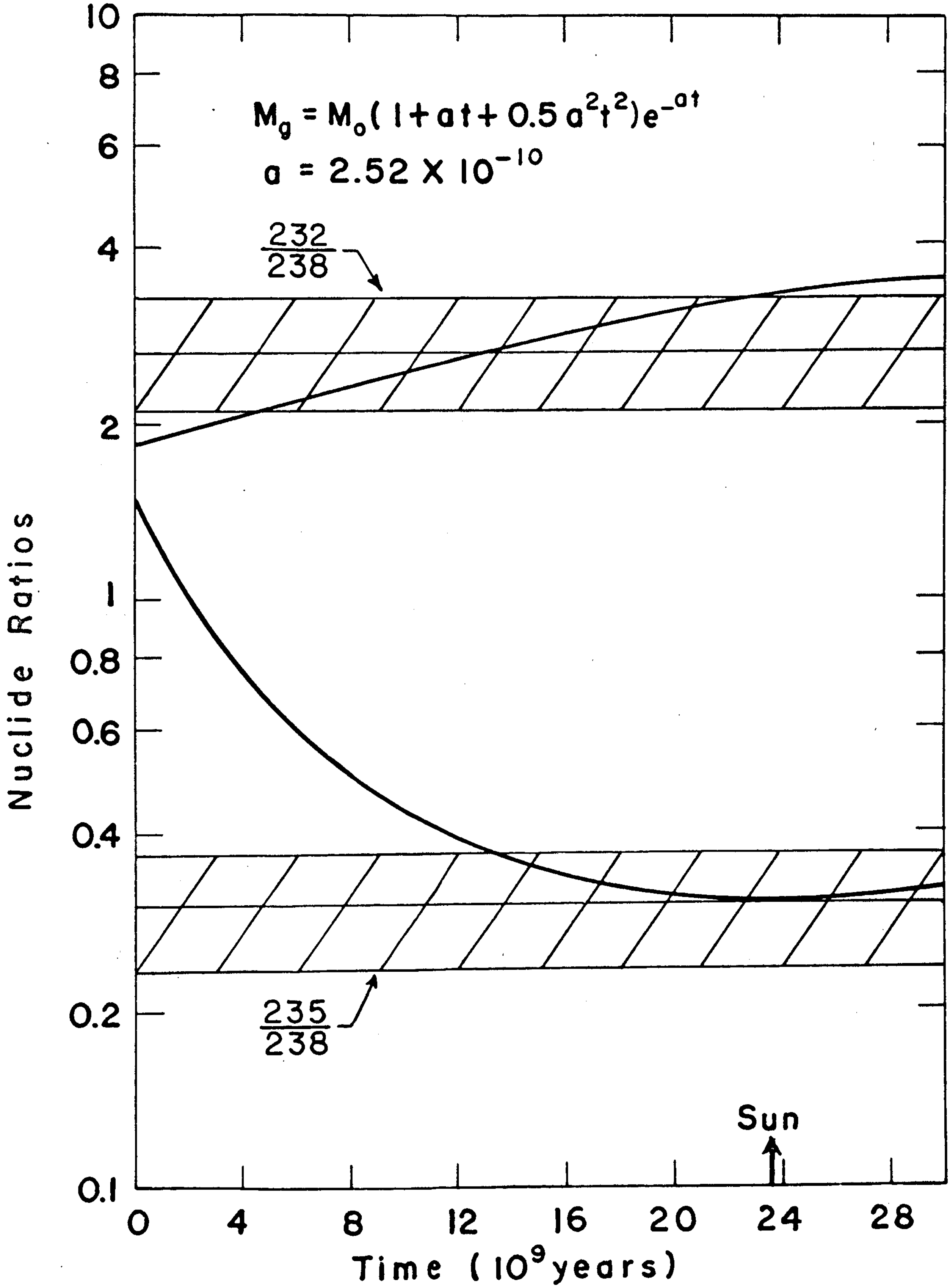

$M_g = M_0 e^{-at}$
$a = 2.996 \times 10^{-10}$
232/238
235/238
Sun
Nuclide Ratios
Time (10^9 years)
10
8
6
4
2
1
0.8
0.6
0.4
0.2
0.1
0
4
8
12
16
20
24
28

FIGURE 6b



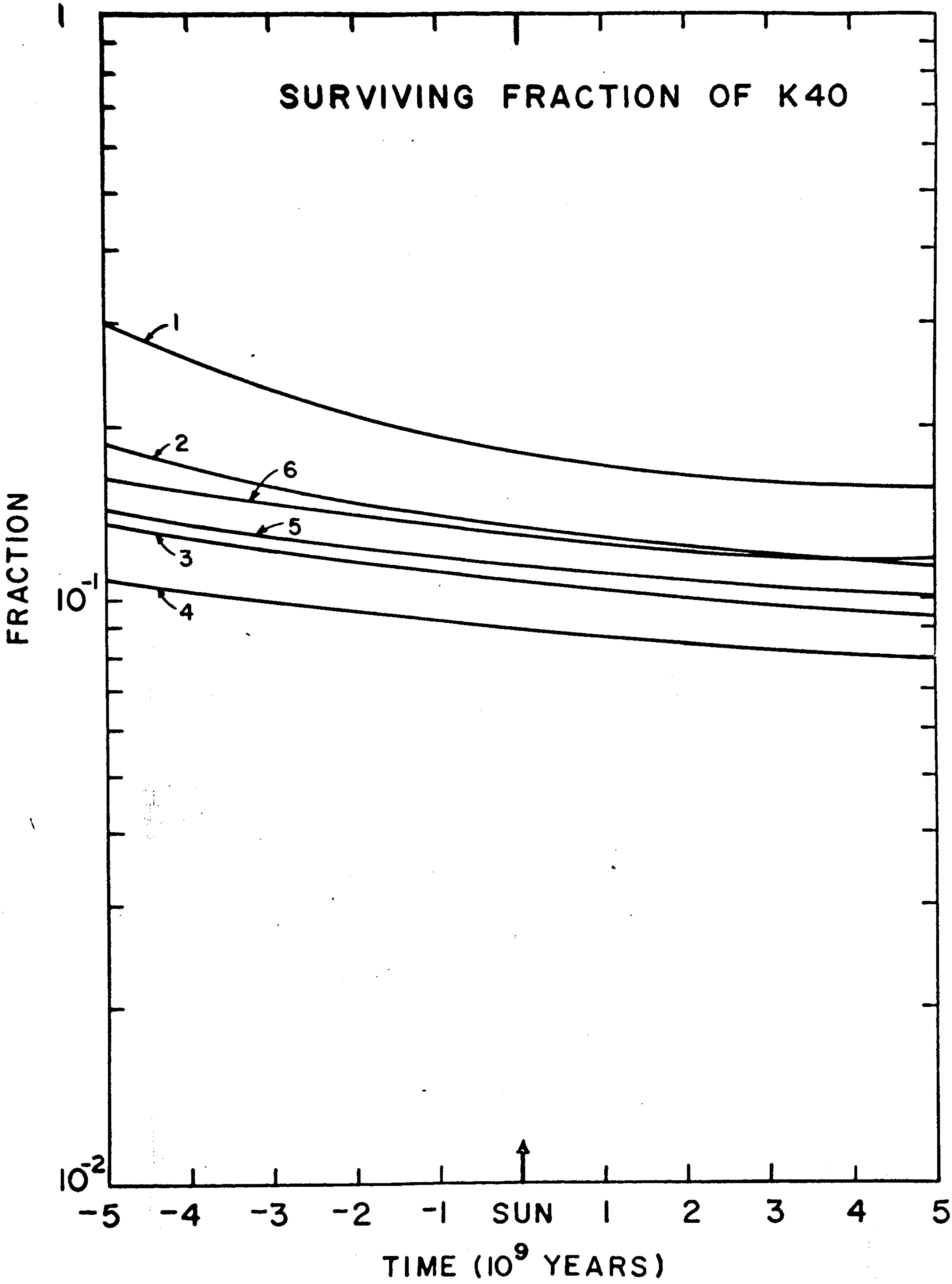
10
8
6
4
2
1
0.8
0.6
0.4
0.2
0.1
Nuclide Ratios
$M_g = M_0 e^{-at}$
$a = 1.198 \times 10^{-10}$
$\frac{232}{238}$
$\frac{235}{238}$
Sun
0
4
8
12
16
20
24
28
Time (10^9 years)

FIGURE 6c



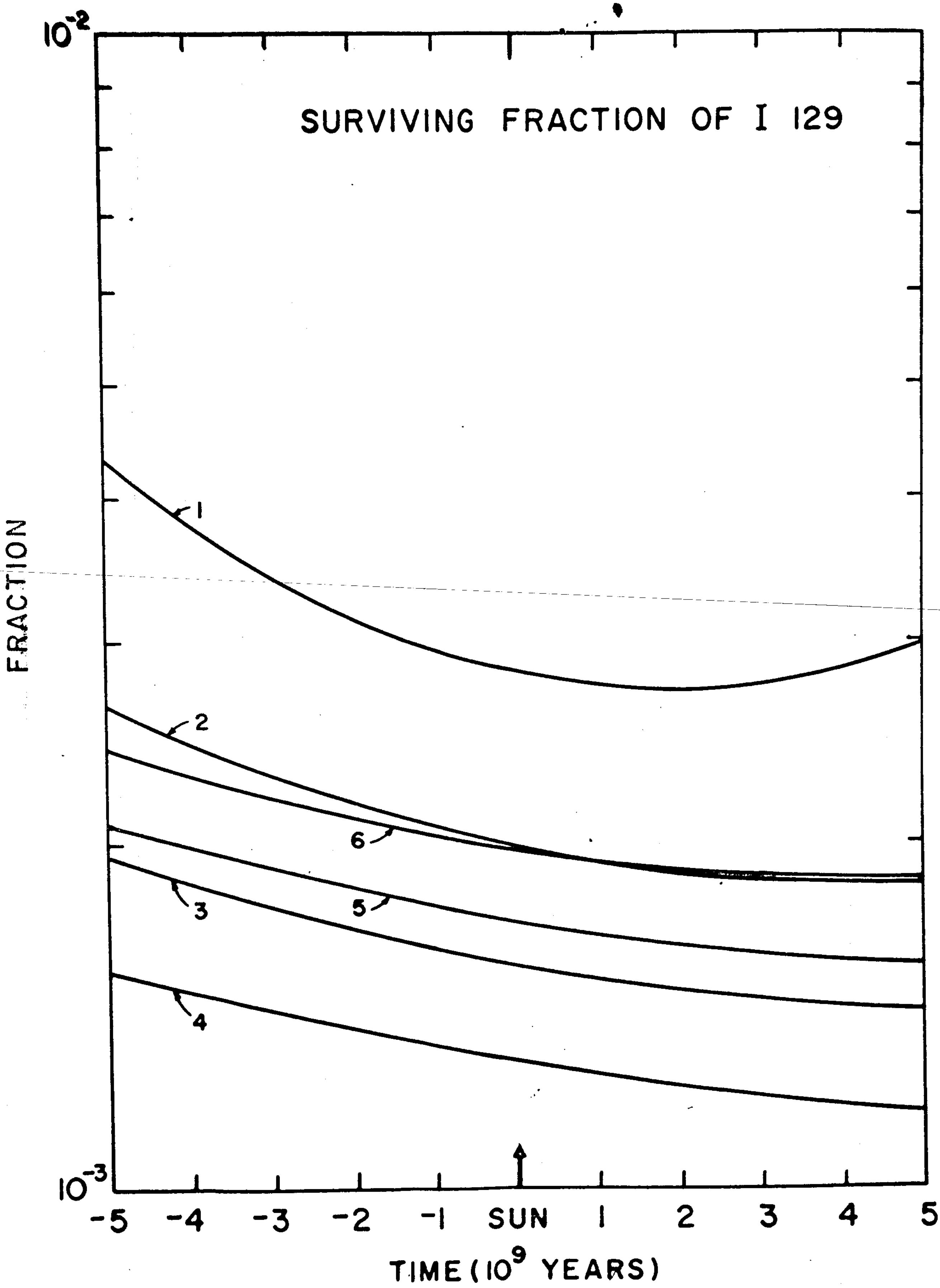
$M_g = M_o(1 + at + 0.5a^2t^2)e^{-at}$
$a = 2.52 \times 10^{-10}$
$\frac{232}{238}$
$\frac{235}{238}$
Sun
Nuclide Ratios
10
8
6
4
2
1
0.8
0.6
0.4
0.2
0.1
0
4
8
12
16
20
24
28
Time (10^9 years)

FIGURE 7



SURVIVING FRACTION OF K40
1
2
6
5
3
4
FRACTION
1
10^{-1}
10^{-2}
−5
−4
−3
−2
−1
SUN
1
2
3
4
5
TIME (10^9 YEARS)

FIGURE 8


SURVIVING FRACTION OF I 129
FRACTION
TIME (10^9 YEARS)
10^{-2}
10^{-3}
-5
-4
-3
-2
-1
SUN
1
2
3
4
5
1
2
3
4
5
6